Mechanics of cell division: Influence of spontaneous membrane curvature, surface tension, and osmotic pressure

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Cell constriction is an important cytokinetic phase preceding division. Before splitting in two daughters, symmetrically dividing cells accommodate theirs duplicated contents into spatially separated compartments defined by a stable fission site located at midcell. Constriction is a non-spontaneous process which involves large membrane deformations at the site of fission, a division route entailing a strong breakage of symmetry in the mother cell. Independently of the details that determine the functioning of the constriction engine, the membrane component responds against elastic deformation by minimizing the energy necessary at every constriction stages (Figure 1). Despite the importance of this knowledge to unveil the details of cell division, the precise mechanical pathways during this process are unknown.

In this work, we address a theoretical study of the mechanics of membrane constriction in a simplified vesicle model under given pressure and tension conditions in the regime of comparable mechanical work due to these variables than to the bending energy. We develop a general method to find approximate analytical expressions for the main magnitudes of a symmetrically constricted vesicle in terms of the spontaneous curvature, the surface tension, and the osmotic pressure.

In terms of the local principal curvatures of the surface, C_1 and C_2, under the osmotic pressure Δp (the inner minus the outer pressure), and under the action of the surface tension Σ, the total energy of the vesicle is given by

$$E_b = \frac{\kappa}{2}\int_\Omega (C_1 + C_2 - C_0)^2 \mathrm{d}A + \int_\Omega \Sigma dA + \Delta p \int_V dV,$$

where κ is the bending modulus, Ω the surface, $\mathrm{d}A$ its element of area, C_1 and C_2 its principal curvatures, and the parameter C_0 is the spontaneous curvature that effectively accounts for possible asymmetries in the surface structure between the inner and the outer sides.

Analytical solutions are obtained by combining a perturbative expansion for small deformations with a variational approach that was previously demonstrated valid at the reference state of a flaccid vesicle. The theoretical approximate results are compared with the exact solution obtained from numerical computations, getting a good agreement for all the magnitudes calculated including the constriction energy, constriction force, membrane area and volume, and length of the vesicle.

We analyze the effects of the spontaneous curvature, the surface tension and the osmotic pressure in all these magnitudes, focusing especially on the constriction force.

The conditions under which vesicles constrict more easily are determined, obtaining that smaller constriction forces are required for positive spontaneous curvatures, low or negative membrane tension and hypertonic media. Conditions for spontaneous constriction at limiting constriction force are also determined.

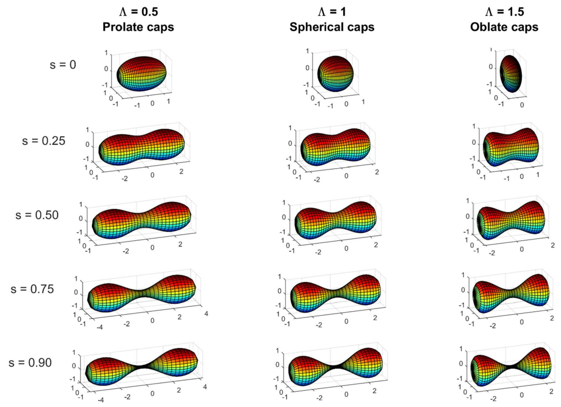

Figure 1: Shapes at various constriction stages ($s = 0$; $s = 0.25$; $s = 0.5$; $s = 0.75$; and $s = 0.9$) for initially unconstricted prolate, spherical, and oblate vesicles.

All these results contribute to a better quantitative understanding of the mechanical pathway of cellular division, and could assist the design of artificial divisomes in self-actuated microsystems.

[1] Víctor Almendro-Vedia, Francisco Monroy, and Francisco J. Cao. *Mechanics of Constriction during Cell Division: A Variational Approach*. Plos ONE. **8**, e69750 (2013).

[2] Víctor Almendro-Vedia, Francisco Monroy, and Francisco J. Cao. *Analytical results for cell constriction dominated by bending energy*. Phys. Rev. E. **91**, 012713 (2014).

[3] Elena Beltrán-Heredia, Víctor Almendro-Vedia, Francisco Monroy, and Francisco J. Cao. *Mechanics of cell division: Influence of spontaneous membrane curvature, surface tension, and osmotic pressure*. (Submitted 2017)