Boussinesq convection in a gas of hard disks, induced by inelastic lateral walls and gravity

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We study a gas of smooth hard disks enclosed by four walls that delimit a rectangular region. The gas is subject to a constant gravitational field $\mathbf{g}$. The horizontal (i.e, perpendicular to gravity) pair of walls is provided with a kinetic energy source in its lower (in the sense of gravity) wall. The upper wall is far away and does not provide any energy input. We label the other two walls as 'lateral' walls. The lateral walls do not provide any energy input. When the lateral walls are elastic (wall-particle collisions preserve kinetic energy), the system shows a steady base state that is hydrostatic [1]. On the contrary, if the lateral walls act as an energy surface sink, the unidimensional steady state becomes obviously two-dimensional and (not trivially) always convective [2]. Furthermore, this type of convection has not been analyzed in the previous literature on fluid convection [3]. We show that, theoretically, this convection may be described in the context of the Boussinesq equations [4], adapted to take into account the kinetic energy loss upon wall-particle collisions.

Optionally, we consider also inelasticity in particle-particle collisions. This case is relevant since, as it is known, for macroscopic particles kinetic energy is partially transferred into the dynamics internal degrees of freedom; for instance, into thermal movement of the molecules that are the constituents of the disk material [5]. Once it is transferred, this kinetic energy is not recovered again into the grains dynamics. *Granular gases* can be produced in a straightforward way in laboratory experiments, if, for example, air flow [6] or mechanical vibration [2] are used as energy inputs. In the case of vibrated systems, no interstitial fluid is necessary in order to keep the granular dynamics [7].

But we can also find granular gases in nature. For instance, dust clouds develop as atmospheric aerosols in the troposphere and can travel long distances, crossing entire oceans and continents [8]. Sand storms are also important in reduced-gravity environments, like in the Mars atmosphere, where aided by extremely strong winds they play a major role in the erosion processes. Planets rings can be another granular gas example [9]. In fact, one interesting example of granular gas system is the set of Uranus rings, some of which are dust-free and only composed by larger particles, such as the ϵ-ring (caged by two shepherd moons), with an average particle size of $0.1 - 10$ m [10].

We compare our theory results with laboratory experiments, Monte Carlo simulations of the Boltzmann equation of the granular gas and molecular dynamics simulations. All four independent methods show excellent agreement in the global properties of the new convection. See Figure 1 for a comparison between theory and experiment. We explain in this work the theory details that lead to this agreement with experiments.

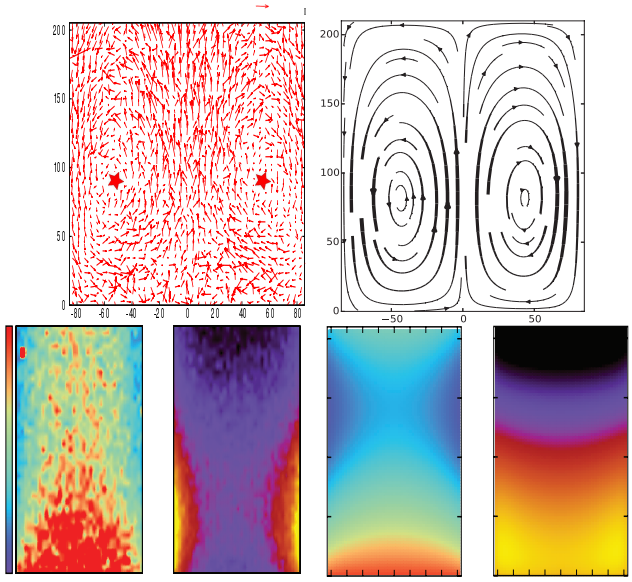

Figure 1: Fields from experiments and theory (left & right columns, respectively). Length unit: particles diameter ($= 1$ mm). $g = 0.016 \times 9.8\ \mathrm{m/s^2}$. Top row: flow field. Bottom row: the corresponding temperature T/m (left) and packing fraction $\nu = n\pi(d/2)^2$ (right) fields. Black stands for lower and read for higher field value. $0 < T \leq 0.2mv_0^2$, ($v_0 = 370\ \mathrm{mm/s}$); $0.02 < \nu \leq 0.15$. Experiments: with $f = 45$ Hz, $A = 1.85$ mm. Theory: coefficient of restitution $\alpha = 0.9$ (both for particle-particle and wall-particle collisions).

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