From quasiperiodic partial synchronization to collective chaos

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Neuronal rhythms are frequently modeled by means of populations of oscillators, capable of exhibiting a wide diversity of complex macroscopic dynamics. A relevant example is quasiperiodic partial synchronization (QPS), a state in which individual units behave quasiperiodically, while global observables oscillate periodically. This disparity of macroscopic and microscopic rhythms is shared by collective chaos, in which individual oscillators behave irregularly, usually reflecting microscopic chaotic dynamics.

As shown in [1], populations of limit-cycle oscillators may also display pure collective chaos, a state without orbital instability at the microscopic level. In this contribution we report on a route from QPS to pure collective chaos in an analytically tractable population of inhibitory neurons with delayed interactions.

Our model consists of an ensemble of $N \gg 1$ globally coupled quadratic integrate-and-fire (QIF) neurons. Thus, each neuron is described uniquely in terms of its membrane potential V_j, which obeys a quadratic ordinary differential equation:

$$\dot{V}_j = V_j^2 + \eta_j + J\, r_D. \tag{1}$$

Here η_j determines the dynamics of the uncoupled neuron ($J = 0$). The reset conditions are set at infinity: every time t_j^k ($k = \ldots, 0, 1, 2, \ldots$) V_j reaches ∞, neuron j is said to fire and its potential is reset to $-\infty$. The term $r_D \equiv r(t-D)$ in Eq. (1) is the delayed firing rate of the population. It can be conveniently defined taking the limits $N \to \infty$ and $\tau_s \to 0$ from this expression:

$$r_D = \frac{1}{N\tau_s} \sum_{j=1}^{N} \sum_{k} \int_{t-D-\tau_s}^{t-D} \delta(t' - t_j^k)\, dt' \tag{2}$$

Nontrivial collective dynamics, in the form of QPS and collective chaos, are shown in Fig. 1. The ensemble consists of identical neurons ($\eta_j = 1$) with inhibitory delayed coupling ($J < 0$, $D > 0$). Note that the observed collective chaos is purely macroscopic since the firing order of the individual neurons is preserved. This is due to (a) the mean-field character of the interactions, and (b) the first order kinetics of the neurons.

In order to find out at which parameter values nontrivial dynamics emerge, we used the techniques in [2] permitting us to reduce the dynamics from N to only two equations for the firing rate (r) and and the mean membrane potential (v):

$$\dot{r} = \frac{\Delta}{\pi} + 2\, r\, v, \tag{3a}$$

$$\dot{v} = v^2 + \bar{\eta} + J\, r_D - \pi^2 r^2, \tag{3b}$$

This strong reduction of dimensionality is possible after assuming that the η_j are distributed according to a Lorentzian distribution $g(\eta) = (\Delta/\pi)[(\eta - \bar{\eta})^2 + \Delta^2]^{-1}$.

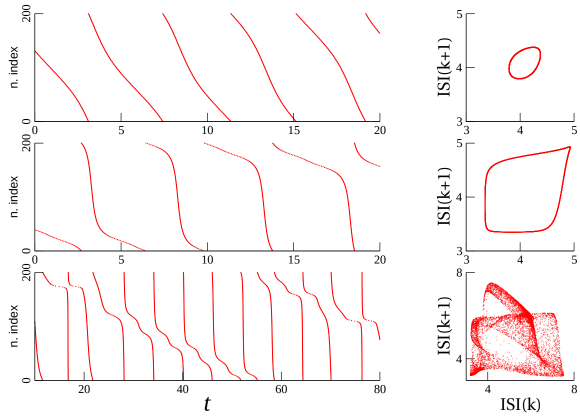

Figure 1: Upper and middle rows show quasiperiodic partial synchronization for a delay $D = 2.5$, and couplings $J = -1.65$ and -1.85, respectively. Lower row shows pure collective chaos for $D = 3$ and $J = -3.8$. The left panels are raster plots of 200 neurons, while the right panels are return plots of the consecutive interspike intervals $\mathrm{ISI}(k) = t_j^{k+1} - t_j^k$ for an arbitrary neuron.

The conclusions of our study [3] are: (i) With QIF neurons, QPS and collective chaos are only possible for inhibitory coupling and certain delays; (ii) the route from QPS to collective chaos is the classical period-doubling cascade; (iii) pure collective chaos is robust under small heterogeneity in the parameters.

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