Null models for social hierarchical structure

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The set of active social relationships between an individual and their acquaintances can be represented as an ego-network, that is a weighted star network where the link weights represent relationship strength or emotional closeness. To maintain and monitor social relationships is costly, and ego-network size is limited by attention and cognitive constraints. They have a typical size of approximately 150, a limit known as the Dunbar number [1]. Moreover, these network are also structured in concentric *layers* of size k_r. These layers are increasingly large sets of ties of decreasing emotional closeness [2]. The layers form a nested hierarchy, where the cumulative sizes of consecutive groups, n_r, follow a preferred scaling ratio of approximately $1/3$, resulting in a sequence of typical group sizes of 5, 15, 50 and 150. This hierarchical structure appears to be a fundamental organizational principle of human groups, and has been confirmed in online games, online social networks and telephone call detail records.

Our work proposes a micro-canonical and grand-canonical ego-network statistical ensembles that reproduce qualitatively the hierarchical structure. The networks are represented as integer-valued weighted networks made of distinguishable links, in the line of reference [3]. We simplify the problem by considering that ties belonging to a given layer have a constant cost, and postulate an abstract social capital or resource, s, that is spent in placing the links into the different layers. The micro-canonical ensemble models a single actor with fixed degree k and total social resource s, and the grand-canonical ensemble models a social group of many actors with given average degree, $\langle k \rangle$, and social resource, $\langle s \rangle$. We prove that both ensembles are equivalent in the thermodynamic limit, where the layer degrees k_r are Poisson variables. We investigate the conditions for a constant group size scaling, arriving to our main result, which states that a constant group-size scaling is possible only if the cost difference is equal across all layers. Finally, we have fitted the ensembles to the Reciprocity Survey (RS) dataset, where 84 university students were asked to value their relationship in a scale from 0 to 5 [4]. The RS dataset shows a hierarchical layer structure, which is well represented by the typical ego-network instances generated by the models, as can be seen in the figure.

In our framework, when the layer costs are equispaced, and for typical given values of social resource, and number of ties, the configuration space region with the maximum number of microstates is found around layer degree values $\{k_r\}$ that verify the scaling relationship. Assigning these layers a constant cost is a convenient simplification which suggests an interesting possibility: That the prevalence of a hierarchical social structure may not arise from a specific organizational principle, but rather appear as a combinatorial effect.

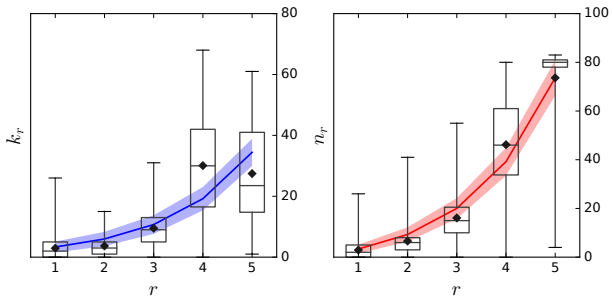

Figure 1: Layer size (left) and layer cumulative size (right) distributions. The empirical RS distributions are represented by the black box-plots, where the box comprises the second and third quartiles, separated by the median line. The whiskers extend to the full distribution domain, and the averages are represented by the black diamonds. The colored lines join the grand-canonical ensemble averages, $\langle k_r \rangle$ (blue, left) and $\langle n_r \rangle$ (red, right), and the dotted lines comprise one standard deviation.

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