Mutualism through the k-core lens

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Mutualistic communities play an important role in biodiversity preservation. They are modelled as bipartite networks that share a set of global statistical properties such as nestedness and modularity. Local measurements of centrality and degree help to rank species and their relative importance for network resilience. However, there is not a unified theoretical framework to explain the relationships among all these parameters.

In this work, we discuss how a classical graph analysis tool, the *k-core* decomposition, sheds light on the understanding of mutualistic networks. It offers a new vision of the structure both at local and global scales. We define three magnitudes using the *k properties*. First, *k-radius* describes network compactness. Second, *k-degree* maps the degree onto a finer grain distribution. Finally, *k-risk*, an index to identify species whose disappearance poses a greater threat to the network. They provide efficient rankings to evaluate robustness.

This procedure was first described by Seidman in 1983 to measure local density and cohesion in social graphs [1]. The *k-core* of a graph G is a maximal connected subgraph of degree k. That means that each node is tied to at least k other nodes in the same subgraph. The most common algorithm to perform the *k-core decomposition* prunes links of nodes with degree equal or less than k. The process starts removing links from nodes with one link until all the remaining nodes have two or more. Nodes that become isolated are the 1-$shell$. Then it continues with $k = 2$, and so on. After performing the *k-decomposition*, each species belongs to one of the *k-shells*. The *m-core* includes all nodes of m-$shell$, $m + 1$-$shell$...

Based on the decomposition, we define three *k-magnitudes*. In order to quantify the distance from a node to the innermost shell of the partner guild, we define k_{radius}. The k_{radius} of node m of class A is the average distance to all species of the innermost shell of guild B. We call this set C^B

$$k^A_{radius}(m) = \frac{1}{\mid C^B \mid} \sum_{j \in C^B} dist_{mj} \qquad m \in A \qquad (1)$$

where $dist_{mj}$ is the shortest path from species m to each of the j species that belong to C^B. The minimum possible k_{radius} value is 1 for one node of the maximum shell directly linked to each one of the maximum shell set of the opposite guild.

To obtain a measure of centrality in this *k-shell* based decomposition, we define k_{degree} as

$$k^A_{degree}\left(m\right) = \sum_j \frac{a_{mj}}{k^B_{radius}\left(j\right)} \quad m \in A, \forall j \in B \qquad (2)$$

where a_{mj} is the element of the interaction matrix that represents the link, considered as binary. If the network is weighted, a_{mj} will count as 1 for this purpose if there is interaction, 0 otherwise. So, $k_{degree}(m)$ is a weighted $degree$ where each node i linked to m adds the inverse of its $k_{radius}(i)$. Generalists score high k_{degree}, whereas specialists have only one or two links and so lower k_{degree}.

Finally, we introduce k_{risk} as a way to measure how vulnerable is the network as a whole to the loss of a particular species:

$$k^A_{risk}\left(m\right) = \sum_i a_{mi} \left(k^A_{shell}\left(m\right) - k^B_{shell}\left(i\right)\right) \qquad (3)$$

with $\quad m \in A, \forall i\, inB, k^B_{shell}\left(i\right) < k^A_{shell}\left(m\right)$

In an intuitive way, if we remove one species that has many links with species of lower *k-shells*, those nodes are in high risk of being dragged by the primary extinction. On the other hand, the extinction is much less dangerous for species of higher *k-shells*, because they enjoy more redundant paths.

The *k-core decomposition* offers a new topological view of the structure of mutualistic networks. We have defined three new magnitudes to take advantage of their properties. Network compactness is described by $\overline{k}_{radius}$, a measure of average proximity to top generalists of the partner guild. This magnitude shows a high correlation with the common measure of nestedness, $NODF$, but only a subset of highly nested networks are compact as well. We think that the concept of compactness has a more direct relationship with the degree distributions of plants and pollinators. From a functional point of view it may suggest that compactness is at least equal or better proxy of network resilience than $NODF$. Second, k_{degree} maps each node's degree onto a more continuous distribution. It has not only information of the number of interactions but also of the distance to the innermost shell of its neighbours. Finally, k_{risk} effectively identifies species whose disappearance poses a greater risk to the entire network.

[1] S. B. Seidman, Social Networks. **5**, 269 (1983).