Dynamics of an ultrathin viscous ferrofluid film under a magnetic field

Enrique Rodríguez-Fernández[1] and Rodolfo Cuerno[1]
[1]Departamento de Matemáticas and Grupo Interdisciplinar de Sistemas Complejos (GISC)
Universidad Carlos III de Madrid, Avenida de la Universidad 30, 28911 Leganés (Madrid)

Ordered patterns are well known to form on the surfaces of ferrofluid films in presence of an external magnetic field. This macroscopic pattern formation process has been explained theoretically in terms of the Rosensweig instability [1] and studied numerically through e.g. a combination of the Navier-Stokes and Maxwell equations [2]. For an example of ferrofluid silver, see Fig. 1.

Figure 1: Ferrofluid droplet under a magnetic field, from [3].

Nanotechnology provides a different, challenging context in which a high degree of order, such as is found for these patterns, would be highly beneficial, in order to enhance optoelectronic, catalytic, and/or other material properties [4]. Actually, bottom-up processes by which surfaces can self-organize into highly ordered structures at the nanoscale are relatively scarce. Note that gravity, which is a stabilizing mechanism in the Rosensweig instability, is negligible at such small distances. However, at the same time the interaction between the free fluid surface and the substrate (disjoining pressure, Π), becomes relevant [5]. Here we make a theoretical proposal to produce highly ordered nanopattenrs on the surface of ultrathin ferrofluid layers on suitable substrates under magnetic fields. Specifically, we put forward a continuum model for the dynamics of the layer thickness h, which we derive analytically and integrate numerically.

The derivation of the effective surface equation has been carried out from the following equations, using the boundary conditions sketched in Fig. 2 and notations as in [6]:

$$\partial_t h = -\partial_x \int_0^h u(y)dy, \qquad \text{(Mass conservation)}$$

$$\nabla P = \eta \nabla^2 \vec{v} + \nabla \cdot T^m, \qquad \text{(Linear momentum cons.)}$$

$$\nabla \times \vec{H} = 0 \Longrightarrow \vec{H} = -\nabla \psi, \qquad \text{(Ampere's law)}$$

$$\nabla \cdot \vec{B} = 0. \qquad \text{(Gauss' law)}$$

By employing the lubrication approximation [6, 7], and in the large magnetic permeability limit, the following effective equation is obtained in dimensionless units [8],

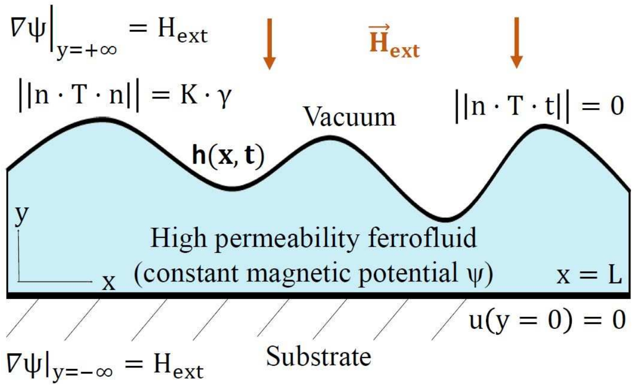

Figure 2: System diagram within boundary conditions.

$$\partial_t h = -\partial_x \left[\frac{h^3}{3} \partial_x \left(\Pi(h) + \frac{L^2 \mu_0 H_{ext}^2}{h_0 \gamma} \mathcal{H}\left(\partial_x h\right) + \partial_x^2 h \right) \right],$$

where $\mathcal{H}(\cdot)$ is the Hilbert transform. For instance, the linear dispersion relation for growth/decay of periodic perturbations with wave vector k to an homogeneous profile $h(x) = h_0$ reads

$$\omega(k) = \frac{h_0^3}{3} \left(\Pi'(h_0) k^2 + \frac{L^2 \mu_0 H_{ext}^2}{h_0 \gamma} k^3 - k^4 \right),$$

which indeed features a narrow interval of unstable Fourier modes (thus, enhanced pattern order) for large enough values of the external magnetic field H_{ext} and suitable surface/substrate interactions. Numerical confirmation of these results in the nonlinear regime of our continuum model will be additionally presented, at which different physical forms for the disjoining pressure will be considered.

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