Inducing Self-Organized Criticality in a network toy model by neighborhood assortativity

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Self-Organized Criticality (SOC) is a paradigm of complex system. "A SOC system is a driven, dissipative system consisting of a medium through which it can propagate disturbances, which cause a modification of the medium, such that eventually, the disturbances are critical, and the medium is modified no more -in the statistical sense" [1]. Empirical examples that have been linked to SOC dynamics are earthquakes, solar flares, neuronal activity, or sand piles among others.

In order to determine the physical properties of these dynamics on complex networks, several models have been proposed. In these models criticality is produced by a "fitness" parameter defined on the nodes or by a rewiring process.

We present a network toy model that is driven exclusively by the network's topology [2]. Starting from a single node, the network grows by adding randomly a new node (with a single link) at each time step. The criticality appears due to a topological stability condition: A node is stable, if and only if its degree is less than or equal to the average degree of its neighbors plus a global constant, C. This local condition is related to a *neighborhood's assortativity*: the tendency of a node to belong to a community (its neighborhood) showing an average property similar to its own. When a node becomes unstable, one of its links is randomly removed and the smallest subnet is deleted. Then the stability conditions of the node and its neighbors are checked iteratively until every node in the network is stable. When all the nodes are stable, a new time step starts. The set of removals performed until every node in the network is stable represents an *avalanche*. The *size* of the avalanche s can be defined as the total number of nodes removed from the network. Fig. 1 depicts an example of the avalanche sizes evolution.

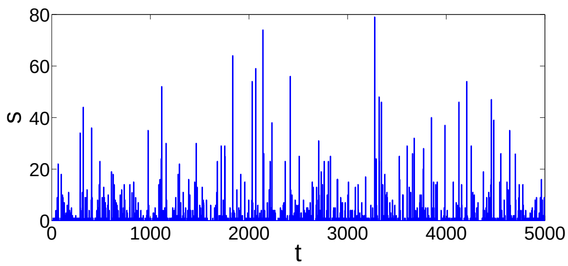

Figure 1: Avalanche sizes evolution during 5000 time units for $C = 2$.

In order to characterize the SOC dynamics we have performed simulations for different values of C. The statistics of the size of avalanches show distributions with similar exponent ($\gamma = 0.8$) to the ones observed in the Olami-Feder-Christensen (OFC) model [3] (see Fig. 2). The statistics of time intervals between two consecutive avalanches shows an exponential behavior. All the distribution plots for different values of C can be collapsed into universal curves.

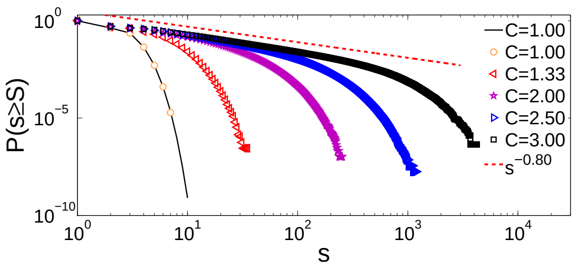

Figure 2: Cumulative normalized event size distribution for $C = 1$ (theoretical result-black line), 1 (orange circles), $4/3$ (red left triangles), 2 (violet stars), 2.5 (blue right triangles), and 3 (black squares); the dashed line with slope $\gamma = -0.8$ is a guide to the eye for the power-law regime.

The probability density function for released energy fluctuations shows the lack of time scales in the correlations. The fit exponent can be compared with the exponent found in the OFC model [3].

The spatial long-range correlations resulting from the criticality have been characterized by means of fluctuation analysis. The results show an anticorrelation in the node's *activity*.

We have performed a theoretical study of this model for the special case of linear chains ($1 \leq C < 4/3$) by means of the *Markov chains*. This statistical approach and the numerical simulations are in complete agreement.

Finally, we have characterized the assortative mixing by vertex degree and the *neighborhood's assortativity*. The assortative mixing by vertex degree is null for this model as for Erdös-Rényi (E-R) or the Barabási-Albert (B-A) models. However, assortative mixing by *neighborhood's average degree* is significantly positive, while it is null for E-R or B-A models. We have found that some real networks exhibit positive neigborhood assortativity and null degree assortativity.

By producing small variations which allow cycles and clusters, the model can become a possible representation for some social organizations, such as corporation hierarchy or population organization.

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[3] F. Caruso, V. Latora, A. Pluchino, A. Rapisarda, and B. Tadić, "Analysis of self-organized criticality in the Olami-Feder-Christensen model and in real earthquakes", Phys. Rev. E **75** (5), 055101-1–4 (2007).