Generic model of population dynamics and transitions between biological interactions

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Population dynamics for interacting species have been modelled with two different type of differential equations: Predator-prey models are usually based on the Lotka-Volterra equations, whereas mutualistic models are used for mutual benefit interactions. Besides the standard functional type II, recently a mutualistic model based on the logistic equation has been introduced, with a simpler functional and with similar state and solutions [1].

In this work we study this model for generic interactions, i.e., for beneficial or detrimental relationships which depend on a set of primary parameters. Conventional models always treat relations among species as static, although it is well known that relations among species can change. Over this model we propose a trade-off equation between the primary parameters that allows the transition from one kind of relationship to another by the evolution of only one secondary parameter. Transition from one relationship to another can be modeled by a change in this secondary parameter resulting in a change in the populations stationary state.

Our starting point is the logistic-mutualistic model developed by [1], whose system of differential equations, for two interacting species, is given by:

$$\begin{aligned} \dot{X}[t] &= X[t]\{r_1 + b_{12}Y[t] - (\alpha_1 + c_1 b_{12} Y[t])X[t]\} \\ \dot{Y}[t] &= Y[t]\{r_2 + b_{21}X[t] - (\alpha_2 + c_2 b_{21} X[t])Y[t]\} \end{aligned} \tag{1}$$

where population of species 1 is represented by the $X[t]$ function and the population of species 2 by the $Y[t]$ function, c_1 and c_2 are positive constants related to the carrying capacity, and the interaction terms α_1 and α_2 can also be considered positive without loss of generality.

The meaning of the different parameters is straightforward: r is the vegetative growth rate and it is related to the interaction of a species with its environment; b_{ij} is the benefit gained by the species i from the interaction with species j in the mutualistic model. These two terms together can be considered as an *effective growth rate*, $r_{ef,i} = r_i + b_{ij}N_j$. On the other hand, α is the term of biological *brake* in the Verlhust representation, and it represents the intraspecific competition. Finally, the parameter c is a limiting factor of the interaction "benefit" (competition for the intraspecific benefit); without loss of generality c can be considered positive.

Although this system of equations has been introduced to model population dynamics with mutualistic interactions, we can study the primary parameters constraints for which this system might model other biological interactions. We can generalize the interaction term b_{ij} considering that a positive value means a benefit and a negative value means a detriment in its effective growth rate due to the interaction with species j.

In this generic model of population dynamics with arbitrary interactions, there are two relevant primary parameters: one that indicates the type of relationship with the environment (r), and another that specifies the relationship with the partner (b_{ij}). The effective growth rate is the sum of both rates.

According to the sign of b_{ij} the resulting biological interaction between two species can be mutualistic (b_{12} and b_{21} positives) or antagonistic (b_{12} or b_{21} negative). Therefore, taking into account the sign for both parameters we find the standard biological relationships: ***i) facultative mutualism***, when $r_{1,2} > 0$, $b_{12} > 0$ and $b_{21} > 0$, i.e., both species obtain net benefits from their environment and from their partners; ***ii) obligate mutualism***, when $r_{1,2} < 0$, $b_{12} > 0$, and $b_{21} > 0$; ***iii) predator-prey***, when $b_{12} > 0$ and $b_{21} < 0$, an interaction of type "species 1 predates/parasitises species 2"; in this case, to avoid prey extinction $r_2 > 0$; ***iv) competition***, when $b_{12} < 0$ and $b_{21} < 0$, and then $r_{1,2} > 0$.

Considering that an increment in either r and b_{ij}, a way of increasing the species population, the improvement of one of them should involve a detriment of the other one. Following this idea we propose a simple trade-off equation for the evolution of the primary parameters, r and b (as a competition model, associated to principle of competitive exclusion [2]):

$$\begin{aligned} \dot{r}[t] &= r[t](a - d \cdot r[t] - e \cdot b[t]) \\ \dot{b}[t] &= b[t](f - g \cdot b[t] - h \cdot r[t]) \end{aligned} \tag{2}$$

where we can assume all the constants positive, that is, $a, d, e, f, g, h > 0$, and then the meaning of these secondary parameters is as follows: a and f are intrinsic growth rates, d and g are limiting parameters (equivalent to the carrying capacities), and e and h are actually the trade-off parameters. It is important to see that the meaning of this growth, limiting and trade-off is related to the set of primary parameters and not to the populations of the species.

This simple model has allowed us to perform transitions from one relationship to another by shifting only one parameter in the dynamical model of the parameters. These types of transitions between different relationships appear frequently in nature. Without considering the evolutionary process, this trade-off model is a simple example of a possible way of transition between different dynamical regimes. The stochastic simulations have shown the stability of the different regimes and the transitions even when they have not been developed in a quasi-static way.

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[1] J. García-Algarra et al., Journal of Theoretical Biology **363**, 332 (2014).

[2] J. Murray. Mathematical biology (1993).