Fluctuations of the Lyapunov exponents in chaotic extended systems

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Finite-time Lyapunov exponents of generic chaotic dynamical systems fluctuate in time. These fluctuations are due to the different degree of stability across the accessible phase-space. A numerical study [1] has revealed that the diffusion coefficient D of the Lyapunov exponents (LEs) exhibits a non-trivial scaling behavior, $D(L) \sim L^{-\gamma}$, with the system size L.

For chaotic dissipative systems, we show that the wandering exponent γ can be expressed in terms of the roughening exponents associated with the corresponding "Lyapunov-surface". Our theoretical predictions are supported by the numerical analysis of several spatially-extended systems [2]. In particular, we find that the wandering exponent of the first LE is universal: in view of the known relationship with the Kardar-Parisi-Zhang equation, γ can be expressed in terms of known critical exponents (see typical example in Fig. 1). Furthermore, our simulations reveal that the bulk of the spectrum exhibits a clearly different behavior and suggest that it belongs to a possibly unique universality class, which has, however, yet to be identified.

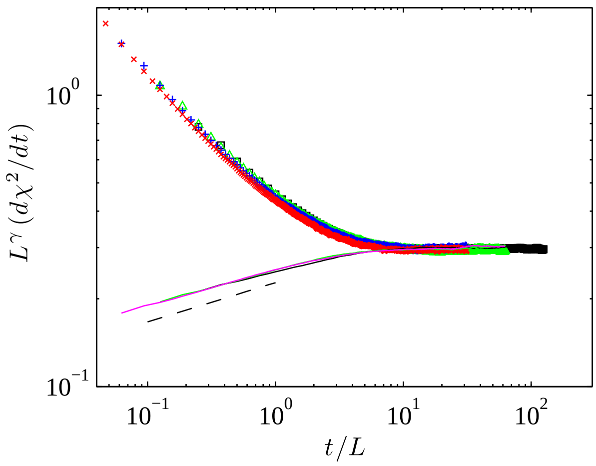

Figure 1: Scaled FTLE fluctuations in a chain of coupled Hénon maps. The exponent γ is set equal to $\gamma = 0.865$. The solid curves (which correspond to $L = 40$, 80, and 160) have been obtained for the 0-norm. The various symbols (squares, triangles, pluses, and crosses correspond to $L = 40$, 80, 160 and 320, respectively) are obtained by using the Euclidean norm of the Gram-Schmidt LVs. The dashed line corresponds to a power law growth $(t/L^z)^\nu$ with $\nu = 0.135$ and $z = 1$.

In contrast, in generic Hamiltonian lattices (see example in Fig. 2) the diffusion coefficient of the maximum Lyapunov exponent diverges in the thermodynamic limit [3]. We trace this back to the long-range correlations associated with the evolution of the hydrodynamic modes. In the case of normal heat transport, the divergence is even stronger, leading to the breakdown of the usual single-function Family-Vicsek scaling ansatz. A similar scenario is expected to arise in the evolution of rough interfaces in the presence of a suitably correlated background noise.

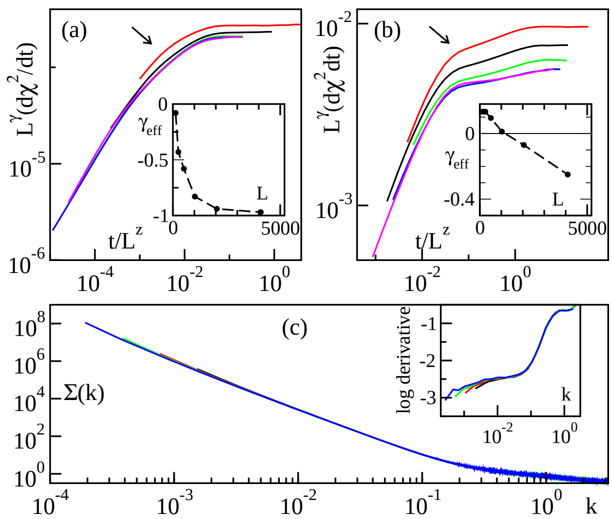

Figure 2: Scaled FTLE fluctuations for the Φ^4 (panel a) and FPU (panel b) models, according to the theoretical ansatz, for sizes $L = 256, 512, 1024, 2048, 4096$. The derivative $d\chi^2/dt$ is considered because of the faster temporal convergence to the asymptotic results. The optimal collapse of the data sets for the larger system sizes is achieved setting $\gamma = -0.97$ and $z = 2.15$ in (a) and $\gamma = -0.25$ and $z = 1.45$ in (b). The insets show the convergence of γ, determined by comparing the sizes L and $L/2$. In panel (c) the structure factor of Lyapunov-vector surface the FPU model is plotted for $L = 4096$, 8192, 16384, and 32768 (see black, red, green, and blue lines). The logarithmic derivative is plotted in the inset, adopting the same color coding.

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