Quantum Monte Carlo approach to fermion ordering in one-dimensional traps

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Since the beginning of the century, experimental progress with cold atomic gases has triggered the realization of a growing number of studies where purely quantum phenomena appear. In particular, the use of traps allows to realize systems that behave as purely one- or two-dimensional ones due to the aspect ratio of the trap used [1]. The rich phenomenology of those low-dimensional quantum systems has been extensively studied both theoretically and experimentally in the last years. In particular, experiments with a few fermions trapped in a one-dimensional harmonic potential have allowed the study of fermionic clusters in one-dimension where the strength of the interactions among fermions is controlled via a Feshbach resonance [2].

A cluster formed by $N_\uparrow$ spin-up plus $N_\downarrow$ spin-down fermions of mass m can be described by a strictly 1D Hamiltonian:

$$H = \sum_{i=1}^{N_\uparrow+N_\downarrow}\left[\frac{-\hbar^2}{2m}\Delta+\frac{1}{2}m\omega^2 z_i^2\right]+g_{1D}\sum_{i=1}^{N_\uparrow}\sum_{j=1}^{N_\downarrow}\delta(z_i-z_j).$$

where the coupling among fermions, g_{1D}, is related to the 1D scattering length of the pair of atoms with unequal spins, a_{1D}, through $g_{1D}=-2\hbar^2/ma_{1D}$.

We have solved Schrödinger equation for systems of few fermions interacting through the Hamiltonian above both in the balanced $N_\uparrow = N_\downarrow$ and unbalanced $N_\uparrow = N_\downarrow + 1$ cases restricting ourselves to repulsive $g_{1D} > 0$ interactions. The properties of the system's ground state are obtained by using the diffusion Monte Carlo (DMC) method [5], which is a stochastic method whose accuracy relies on the election of the *trial* function, some initial guess of the actual ground state wave function [3, 4].

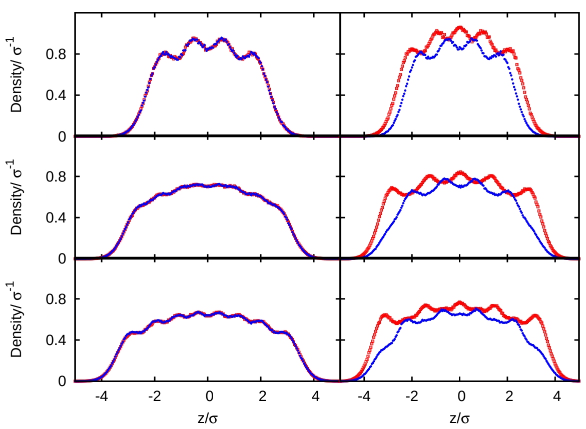

Figure 1: *Normalized density profiles for fermion clusters for the 4+4 balanced case (left) and unbalanced 5+4 case (right) and $g_{1D} = 0$ (top), 10 (center) and 50 (bottom).*

We have obtained the energy of the system, which results compatible with previous estimates [5], and the density distribution for each specie within the trap (See Fig. 1). We observed that while in the balanced case density distributions for both species coincide, in the unbalanced one, maxima for each specie appear at alternate positions. As this may appear as a hint of a dominance of an antiferromagnetic ordering of spins, we checked the orderings for each cluster and realized that the probability to have AF ordering decreases with the number of fermions in the cluster (See Fig. 2). Indeed, when compared to the probabilities when spins are placed in a purely random ordering, the interaction plays a minor role, and it is mostly determined by the configurational entropy of each cluster. Exactly the same can be said of a pure ferromagnetic state.

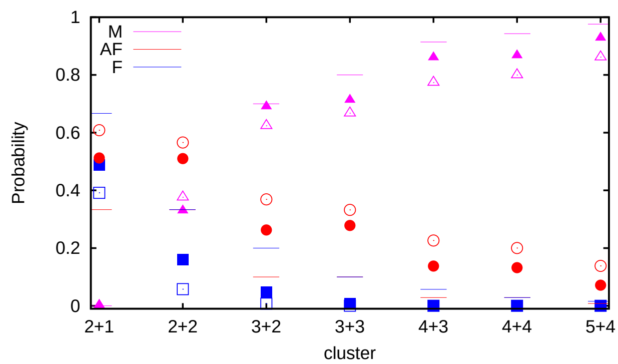

Figure 2: *Fraction of spins in ferromagnetic (F, in blue), antiferromagnetic (AF, in red) and mixed ordering (M, in pink) for a purely random ordering (horizontal line), and the result of our DMC calculation for $g = 0$ (open symbols) and $g = 50$ (full symbols). Note the very particular case of the 2+2 cluster where the three cases have the same probability.*

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