Synchronization of coupled chaotic networks with delay and time-varying connectivity

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In this work we investigate the effect a time varying coupling topology on the synchronization properties of weakly chaotic systems [1]. We study the stability of synchronization for chaotic maps interacting on networks whose topology fluctuates with a characteristic time-scale T_n and whose links bear a large time-delay T_d. We consider an interaction network of coupled chaotic maps with a single coupling delay, T_d. The system is composed by N classical units, characterized by a single degree of freedom $u_i(t)$ and time $t \in N$, whose evolution is given by:

$$u_i(t+1) = (1-\epsilon) f(u_i(t)) + \epsilon \sum_j G_{ij}(t) f(u_j(t-T_d)), \quad (1)$$

where $\epsilon \in [0,1]$ is a real parameter which measures the strength of the interaction, T_d, is the coupling delay and $f : [0,1] \mapsto [0,1]$ is a chaotic map. The coupling topology is encoded in the network adjacency matrix G. The only requirement at this point for the coupling weights is to satisfy a stochasticity condition, such that the *existence* of a synchronized solution $u_i(t) = s(t)$ is guaranteed.

$$\sum_j G_{ij}(t) = 1, \quad (2)$$

for all i and all times t. Nonetheless, this does not inform us about the stability of such synchronized state.

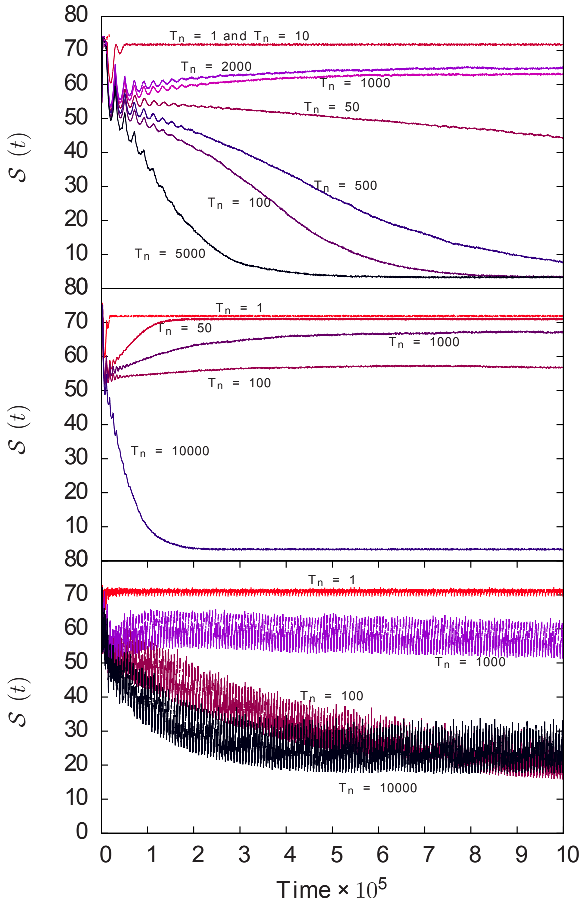
To construct our networks, we consider a fixed directed 1D ring of N sites, where the only non-zero entries are $G_{i,i+1}$ for $i = 1, \ldots N$-1, and $G_{N,1}$. Then, we add to the network $\langle pN \rangle$ *shortcuts*, with $p \in [0,1]$. That is, for every node we establish a directed link to another randomly chosen node with probability p. Every T_n time steps, this procedure is used to sample a new adjacency matrix G.

As a measure of the (zero-lag) synchronization in the network, we have chosen the logarithm of the spatial deviation, σ, of the unit states over the network nodes (for a perfectly synchronized state, $\mathcal{S} \to +\infty$):

$$\mathcal{S} \equiv -\ln(\sigma), \quad (3)$$

We have studied the spectra of the instantaneous networks, and that of the time averaged adjacency matrix, concluding that the mean field eigengap does not describe the synchronization stability correctly. Thus, we have undertaken a computational study of our system.

The figure shows our results for the average time-trace of the synchronization level of several systems on SW networks: A Bernoulli system with $N = 40$, $p = 0.5$ and $\epsilon = 0.7$ (top) and $p = 0.8$ and $\epsilon = 0.47$ (center), and a Logistic system with $N = 40$, $p = 0.5$ and $\epsilon = 0.4$ (bottom). The parameter choices are such that the intantaneous networks are on average non-synchronizing, synchronizing and non-synchronizing, respectively.



We found the stability of the synchronized state to be strongly affected by the interplay between the time-scale of the delayed interactions, T_d, and that of the network fluctuations, T_n. For the fast-switching regime, $T_n \ll T_d$, we obtain a strong enhancement of the synchronizability of the network. Even when we restrict our topology fluctuations to only explore those networks for which synchronization is not stable under static conditions, we can obtain almost-sure synchronization under fast enough fluctuations. This result is in qualitative agreement with the fast switching approximation for diffusively coupled systems [2]. For $T_n \sim T_d$ we observe a severe reduction in synchronizability, which is recovered as we increase $T_n > T_d$. Nonetheless, for $T_n \gg T_d$ the system will nearly always desynchronize.

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