Front interaction induces excitable behavior

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Excitability is a concept that originally comes from biology, inspired by the behavior of neurons and heart cells. An excitable system is characterized by exhibiting a stable steady behavior, while responding to perturbations (e.g. external stimuli) in two different ways: for stimuli below a certain threshold the system decays exponentially to the steady state, while for stimuli above threshold it exhibits a non-trivial excursion in phase space before decaying back to the steady state.

In the case of spatially extended systems, one has first the case of local excitability, what leads to the propagation of travelling waves (autowaves). In addition, spatially extended systems can support local transient excitations in which just a part of the system is excited, namely if the system exhibits localized structures, that become unstable through certain global bifurcations. All these mechanisms, local excitability and excitability of localized structures, have in common an oscillatory instability in phase space, namely one in which a non-zero amplitude limit cycle is destroyed when changing a parameter. The excitable trajectory would follow the remnants of the cycle and ultimately return to the steady state.

In the context of cellular biology, transient localized excitations, also called patches, have been observed in early stages of cell migration. In [1] an explanation was suggested in terms of a spatially extended model with local FitzHugh-Nagumo dynamics in the excitable regime. However, as pointed out in [2], in cell migration, direct evidence for excitability is lacking.

Here we present an alternative mechanism [3] leading to transient patches that requires neither local excitability, nor oscillatory localized structures, not even the existence of the remnants of a limit cycle, widening the classes of extended systems that can present excitability. In particular we show that only two simple ingredients are necessary: bistability between two homogeneous stable steady states (HSSSs) and spatial coupling allowing for monotonic fronts connecting these two states. We show the existence of a threshold for perturbations of the homogeneous state. Sub-threshold perturbations decay exponentially, while super-threshold perturbations induce the emergence of a long-lived structure formed by two back to back fronts joining the two homogeneous states. While in typical excitability the trajectory follows the remnants of a limit cycle, here reinjection is provided by front interaction, such that fronts slowly approach each other until eventually annihilating. This front-mediated mechanism shows that extended systems with no oscillatory regimes can display excitability.

To illustrate the new excitability mechanism we consider a prototypical model displaying bistability, namely, the Ginzburg-Landau equation for a real field $u = u(x,t)$ in one spatial dimension x

$$\partial_t u = \mu u - u^3 + \partial_{xx} u \; . \tag{1}$$

The system is variational and therefore does not have oscillatory solutions. For $\mu > 0$ the system has two equivalent Homogeneus Stable Steady States (HSSs) $u_\pm = \pm\sqrt{\mu}$ and stable fronts that connect them. The trivial solution $u = 0$ is unstable and plays the role of a separatrix for the local dynamics. There are two possible front solutions, known as kink and antikink, with opposite polarity and a monotonic tanh shape. Fronts with opposite polarity attract each other with a strength that decays exponentially with the front separation, and they ultimately annihilate in a behavior known as coarsening.

The rationale of the excitability mechanism is as follows. While the system is sitting on a HSSS, small localized perturbations decay exponentially. Instead, for perturbations exceeding u(x) = 0 in a wide enough spatial region, part of the system will initially evolve to the other (attracting) HSSS leading to the formation of a pair of kink-antikink fronts. In a second stage the two fronts interact, slowly approaching each other. If the resulting kink-antikink structure is relatively broad this second stage will be long-lived. Finally, in a third stage, kink and antikink annihilate each other and the system returns to the initial HSSS. These structures can be viewed as excitable excursions (see Fig. 1). Further information can be found in [3].

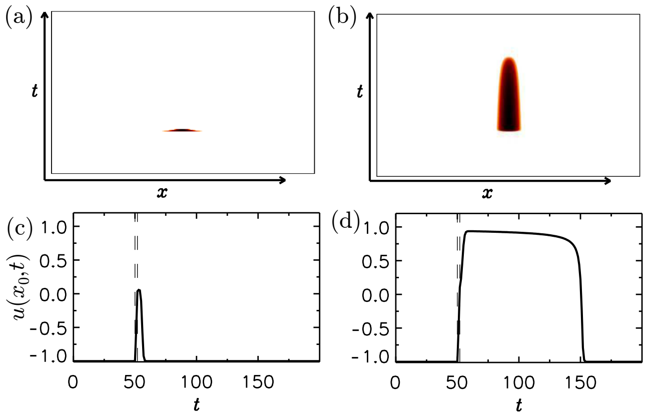

Figure 1: Evolution of $u(x,t)$ after a perturbation on u_-: (a) and (c) correspond to a sub-threshold perturbation; (b) and (d) correspond to a super-threshold perturbation.

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