In-phase, out-of-phase and T/4 synchronization of square waves in delay-coupled non-identical optoelectronic oscillators

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Time delays appear naturally in a variety of physical, biological, chemical and technological systems, and are a source of oscillatory instabilities. When the delay is large compared to the other time scales of the system, square-waves are the dominant solutions for specific values of the parameters. Prototypical systems displaying square-waves are delay-line optoelectronic oscillators (OEOs) which can be modeled by an Ikeda-like equation with feedback proportional to $\cos^2[x(t-\tau)+\Phi]$ where τ is the delay time and Φ a constant offset phase which fixes the operating point [1]. For offset phases $\Phi \in [0, \pi/2]$ feedback damps small perturbations (negative feedback) while for $\Phi \in [-\pi/2, 0]$ amplifies them (positive feedback). Square waves in OEOs with negative feedback have a symmetric duty cycle, namely the duty cycle is half the period. In contrast, for positive feedback square waves have an asymmetric duty cycle [2].

Coupling with delay two oscillators, each with an intrinsic long delay so that it generates square-waves, leads to interesting dynamics arising from the interplay between the intrinsic and the coupling delay times. In particular, two mutually coupled identical OEOs with negative feedback can generate stable in-phase and out-of-phase synchronized square waves with symmetric duty cycle when the ratio between the delay times fulfills suitable conditions [3]. For positive feedback in- and out-of-phase synchronization is also obtained but in this case the duty cycle is not symmetric [4]. In general, the square-waves generated so far with optical and optoelectronic systems have been obtained synchronized in-phase or out-of-phase.

Although in- and out-of-phase are the most common cases of synchronization of two coupled oscillators, they are not the only possible ones. There is a variety of systems for which synchronization at a quarter of the period has been observed. This includes some animal gaits of quadrupeds as walk and jump [5], the limb coordination in crustacean swimming [6], predator-prey cycles [7], human cortical sources [8], neural networks [9] and solar transverse oscillations [10]. Nevertheless, in any of these studies the oscillations have a square-wave shape.

Here by exploring the dynamical regimes that can arise in a prototypical model for mutually delay-coupled OEOs (Fig. 1) we show that stable square-wave pulses synchronized at a quarter of the period do exist in a broad parameter region (Fig. 2). The key point to obtain such $T/4$ solutions is that the two OEO operate with different offset phases, in such a way that the feedback is negative in one and positive in the other, namely the feedback is mixed [11]. Multiple harmonics can coexist. We have also check that thse solutions are robust to mismatch in the delay times up to a few percent. Finally, it should also be emphasized that these $T/4$ synchronized square waves are in the ns time scale, orders of magnitude faster than the systems discussed previously.

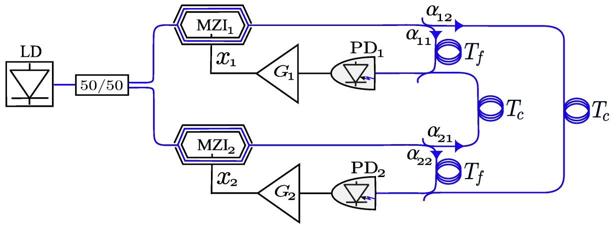

Figure 1: Diagram of the system modeled. Each OEO consists of a Mach-Zehnder interferometer (MZI), a fiber loop with delay T_f, a photodiode (PD) and a RF amplifier (G) whose output modulates an MZI arm. OEOs are fed by a laser diode (LD) and are mutually coupled with delay T_c.

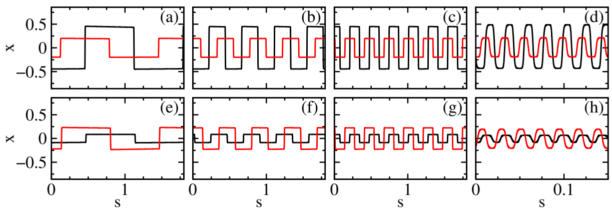

Figure 2: Coexistence of $T/4$ square waves. Fundamental, 1^{st}, 2^{n} and 30^{th} harmonics for $T_c = 60$ns and $T_f = 80$ns are shown in (a)-(d) while (e)-(f) show the corresponding ones for $T_c = 60$ns and $T_f = 40$ns.

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