

Do all glasses empty when turned to the horizontal?

C. Rascón¹, A.O. Parry² and D.G.A.L. Aarts³

¹ GISCS, Departamento de Matemáticas, Universidad Carlos III de Madrid

² Mathematics Department, Imperial College London

³ Physical and Theoretical Chemistry Laboratory, University of Oxford

When a capillary is half-filled with liquid and turned to the horizontal, the liquid may flow out of the capillary or remain in it. For lack of a better criterion, the standard assumption is that the liquid will remain in a capillary of narrow cross-section, and will flow out otherwise. Here, we present a precise mathematical criterion that determines which of the two outcomes occurs for capillaries of arbitrary cross-sectional shape, and show that the standard assumption fails for certain simple geometries, leading to very rich and counter-intuitive behaviour [1]. This opens the possibility of creating very sensitive microfluidic devices that respond readily to small physical changes, for instance, by triggering the sudden displacement of fluid along a capillary without the need of any external pumping.

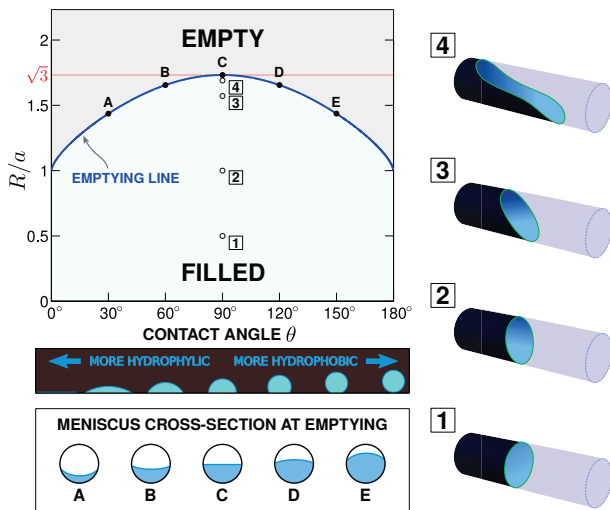


Figure 1: Emptying line for a cylindrical capillary of radius R in units of the capillary length of the liquid a as a function of the contact angle θ of the liquid with the walls. The cross-section of a 3D drop of constant volume is included below to illustrate the interaction of the liquid with a flat wall for different contact angles, from very hydrophilic ($\theta = 0^\circ$) to very hydrophobic ($\theta = 180^\circ$). The emptying line separates the *filled* region (where a meniscus exist) from the *empty* region (where no meniscus exists, which prompts the emptying of the capillary). Plots [1] - [4] show the deformation of the meniscus for $\theta = 90^\circ$ as the emptying line is approached (by increasing the value of R/a), and the appearance of a liquid tongue prior to emptying. Plots A - E illustrate the cross-section of the (infinitely long) liquid tongue at emptying for 5 different contact angles.

[1] C. Rascón, A.O. Parry and D.G.A.L. Aarts, Proc. Natl. Acad. Sci. USA **113**, 12635 (2016)